# DLGF Ratio Study Analysis

## General Procedural Concerns

- 50 IAC 14-8-1 establishes a procedure for a ratio study of using sales prices in comparison with assessment data existing contemporaneously with the submission of the county ratio study. In the case of the Laporte County 2006 ratio study, this was the assessment data in place as of approximately October 2006. How does the DLGF envision a process of the Assessor submitting a ratio study utilizing <u>future</u> assessed value information, ie. that data which will be in effect at the time the assessments are billed to the taxpayer? In the case of Laporte County 2006 pay 2007 assessments and billing information, about one year passed between these two events (submission of the initial ratio study vs. billing information).

  We do not oppose a two-part ratio study process in the future (one study submitted upon completion of assessment functions and another submitted shortly before billing to taxpayers). However, if each county is to perform ratio studies at different junctures in the process, they must be given an opportunity to implement corrective measures even after billing, short of a reassessment.

- Laporte County has backup assessment data from 12-31-06. We are in process of determining the number of assessment changes between that data and the data utilized for billing purposes. We estimate that number to be several thousand 2006 pay 2007 assessment updates. The Auditor billing AV data is often substantially different than that existing at the time of the ratio study. Assessed values are never "final".

- There is no precedence in Indiana to utilize assessed values associated with Auditor billing as the basis of a ratio study under 50 IAC 14. Given that the ratio study is to be completed **before** assessed values are to be rolled to the County Auditor and before tax rates are established, the procedure heretofore necessarily precludes use of Auditor billing AV and necessitates AV in place at the time of the ratio study approval.

- Laporte County is investigating the additional sales utilized in the DLGF study for possible inclusion of invalid transactions.

## 1. Corrections

- Springfield TWP improved residential study. Please see column R, row 2471 and 2474. Zero values exist for the Auditor AV total. Correcting that error results in a COD of 14.29 and a PRD of 1.02; within standard.

- Noble TWP improved residential. The weighted mean is incorrectly calculated by using Auditor total AV vs. 06 assessor total AV. When one uses the total sales price as the correct comparative measure, the statistics are well within standard (median = 1.01, COD = 9.88, PRD = 1.01).

- Center TWP improved commercial. Line 34: Parcel 05 06 35 251 015
  This parcel was also included in the Center TWP vacant commercial study as
  well. At the time of sale, it was a vacant parcel, subsequently improved with a
  bank. Removing this parcel from the Center TWP improved commercial study
  results in statistics well within standard (median = 0.99, COD = 11.48, PRD =
  1.01)

- Michigan TWP improved commercial.

  Lines 155-161. This was a sale of a nursing home. On appeal the purchaser
  provided documentation of substantial personal property and a small amount of
  intangible value (documentation available). The resultant change made was
  equal to the difference between the current assessment and the sales price.
  Exclude (or adjust sales price accordingly).

  Line 221. The Blue Chip Casino purchased this adjacent parcel (parking lot),
  however, land value was established based on other parking lot purchases by
  Blue Chip as well as an effort to establish an overall value for the facility.
  Consider this sale invalid and remove.

  Lines 234-237. Line 237 currently references a summation for lines 222-224
  when in fact it should reference lines 234-236. Correct and leave in study.

  The net effect of these three corrections: Median = 1.00; COD = 9.35; PRD =
  1.02; within standard.

- Scipio TWP vacant residential. Line 687 is actually a two-parcel sale,
  including parcel 62 10 13 300 163. This is somewhat confusing as parcel
  "163" had sold previously and is on the next line as well. Adding this parcel to
  the total assessment for line 687 results in the following statistics: Median =
  0.94; COD = 13.06, PRD = 1.025; within standard.

- Laporte County as whole, combined vacant commercial. Line 58 is not used in
  the calculations for any statistics. Once this error is corrected, the following
  statistics result: Median = 0.96; COD = 26.7; PRD = 1.02; note that the COD
  is still not within standard

## 2.	Spearman Rank Test as a Measure of Assessment Regressivity

IAAO recommends the non-parametric Spearman rank test as a preferred measure of assessment regressivity / progressivity in cases where the sample size is small (20 observations or less). Ties are given mid-ranks. Results:

- Galena TWP vacant RES
  Spearman test results in an insignificant test score; no evidence of assessment regressivity / progressivity.

- Hanna TWP vacant RES
  Spearman test results in a significant test score; evidence of assessment regressivity / progressivity.

- Noble TWP vacant RES
  Spearman test results in an insignificant test score; no evidence of assessment regressivity / progressivity.

- Springfield TWP vacant RES
  Spearman test results in an insignificant test score; no evidence of assessment regressivity / progressivity.

## 3.	Sales chasing

- IAAO Standard on Ratio Studies, Appendix D (p. 56) references that the oversight agency establish some reasonable tolerance in percentage changes between sold and unsold property, such as 3 percent. This recommendation is based on a one-year change in valuation date, such as 2006 to 2007. However, assessed values in 2005 were based on 1999 value whereas assessed values in 2006 were to be based on value as of 2005 (a six year period). If three percent tolerance is provided for a one-year period, what degree of tolerance is applicable for a six year period?

### Use of the Wilcoxan-Mann-Whitney (WMW) Test for Sales Chasing

- IAAO as well as statistical reference texts suggest use of the WMW test as an indicator to test whether two sample emanate from the same population (null hypothesis) or alternatively whether they emanate from different populations. However, the test in this application is envisioned as a one-over test, to examine assessment changes from year-to-year were the result of a sale. When a significant time period exists between valuation dates, when the assessing jurisdiction has engaged in reassessment-type activities between the two valuation dates, and when significant errors were found to exist in prior assessments (and corrected), the test is rendered null and void.

- Reassessment-type activities will tend to invalidate the use of the WMW test as a measure of sales chasing. Between 2002 and 2006, Laporte County has already provided significant documentation of the reassessment-type activities (field reviews, neighborhood delineations, corrections, land basis and value changes) that impacted assessments. In terms of the WMW test, the underlying population is no longer homogenous. Further, the test relies to some degree on homogenous assessment data, ie. the township has minimal variation of property types. Likewise, the test relies on similar ratios of sold and unsold property across the spectrum of property value. To the extent that a township has significant variation of property values, and to the extent that an assessment cycle has experienced significant changes in values since the prior assessment, and to the extent that sold property exchanges at different rates in various stratum of the population, the WMW assumption of a homogeneous population is further violated. Both the Denne analysis and the DLGF analysis by Mr. Schwab to date overlook the significant violations of the basis of the test, resulting in a spurious analysis and irrelevant test. There were significant changes to the population data, above and beyond whether or not the parcel sold.

- Specifically for Michigan TWP, please reference the attached WMW analysis. We compared the 2004, 2005 and 2006 sales data with the 2006 assessed value data existing at the time of the submission of the ratio study to the DLGF. 2006 sales were included as the annual adjustment procedure; these sales were available and utilized in the process to further expand the sample size.

  As an (improperly) combined group, the township as a whole fails the WMW test. However, sold parcels in the Lakefront increased in value by about 85%, whereas inner-city property increased only by about 18%. Further, condo property & Tryon Farm had been grossly under-assessed. Correction of these assessments, changes to the valuation technique and further stratification, led to dramatic increases in assessments. The data shows that condos sold at a rate twice (18% vs. 9%) as high as the inner city area. Likewise, the lakefront properties sold at a slightly higher rate than inner city property as well. When high value condo & lakefront property is more likely to sell than lower value inner-city property, the WMW will yield a spurious test score.

  Combining all three groups into one population violates an important assumption of the WMW test. Further, please review the median % change figures by neighborhood for Michigan TWP. Virtually all neighborhoods have very similar changes.

  Combining these facts leads to one conclusion about the WMW as an overall statistical measure in Michigan TWP: It is not a valid measure or indicator of sales chasing. Once the data is parsed into proper comparative groups, the WMW can not reject a null hypothesis that sold and unsold property groups were treated equally, ignoring all other violations of the test assumptions.

**Galena Vacant RES**

| Parcel | AV | sales price | Rank of AV | Rank of SP | Diff in Rank, sq |
|---|---|---|---|---|---|
| 200325136023 | 9200 | 5500 | 1 | 1 | 0 |
| 200322100017 | 13500 | 23900 | 2 | 4 | 4 |
| 200320100018 | 22200 | 19000 | 3.5 | 2.5 | 1 |
| 200320100019 | 22200 | 19000 | 3.5 | 2.5 | 1 |
| 650233101035 | 25300 | 32000 | 5 | 7 | 4 |
| 200307400015 | 26700 | 34000 | 6.5 | 12 | 30.25 |
| 200307400015 | 26700 | 34900 | 6.5 | 14 | 56.25 |
| 200307400018 | 29900 | 27500 | 8 | 5 | 9 |
| 200307400026 | 32600 | 32500 | 9 | 8.5 | 0.25 |
| 200307400024 | 33300 | 32500 | 10 | 8.5 | 2.25 |
| 200307400034 | 33400 | 37000 | 11 | 17 | 36 |
| 200307400009 | 33500 | 34000 | 12 | 12 | 0 |
| 200307400017 | 33600 | 36000 | 14.5 | 16 | 2.25 |
| 200307400031 | 33600 | 35000 | 14.5 | 15 | 0.25 |
| 200307400031 | 33600 | 30000 | 14.5 | 6 | 72.25 |
| 200307400033 | 33600 | 34000 | 14.5 | 12 | 6.25 |
| 200307400012 | 33700 | 33000 | 17 | 10 | 49 |
| 200307400025 | 33800 | 43000 | 18 | 18 | 0 |
| 200307400029 | 39500 | 49500 | 19 | 19 | 0 |

Count 19      Sum 274

Test Statistic =  $\dfrac{1- (6*T)}{n * (n*n - 1)}$      **-0.240**

The value is not significant.
No evidence of regressivity / progressivity.

**Hanna Vacant RES**

| Parcel | AV | sales price | Rank of AV | Rank of SP | Diff in Rank, sq |
|---|---|---|---|---|---|
| 241824100018 | 12900 | 20000 | 1 | 1 | 0 |
| 241818300025 | 13700 | 42000 | 2 | 10 | 64 |
| 241818300030 | 13700 | 41000 | 3 | 9 | 36 |
| 241807200009 | 21000 | 21500 | 4 | 3.5 | 0.25 |
| 241807200014 | 21000 | 24500 | 5 | 6.5 | 2.25 |
| 241807200006 | 21700 | 21500 | 6 | 3.5 | 6.25 |
| 241807200008 | 21700 | 21500 | 7 | 3.5 | 12.25 |
| 241807200007 | 22400 | 21500 | 8 | 3.5 | 20.25 |
| 241807200012 | 22500 | 24500 | 9 | 6.5 | 6.25 |
| 241805400024 | 33600 | 34500 | 10 | 8 | 4 |

Count 10      Sum 151.5

Test Statistic =  $\dfrac{1- (6*T)}{n * (n*n - 1)}$      **-0.917**

The value is significant.
Evidence of regressivity / progressivity.

**Noble Vacant RES**

| Parcel | AV | sales price | Rank of AV | Rank of SP | Diff in Rank, sq |
|---|---|---|---|---|---|
| 531433200004 | 21000 | 20000 | 1 | 1 | 0 |
| 531408426005 | 23100 | 38000 | 2 | 4 | 4 |
| 531406200023 | 25000 | 25750 | 3.5 | 2 | 2.25 |
| 531433300014 | 25000 | 30000 | 3.5 | 3 | 0.25 |
| | | Count | 4 | Sum | 6.5 |

Test Statistic = $\dfrac{1-(6*T)}{n*(n*n-1)}$   **-0.633**

The value is not significant.
No evidence of regressivity / progressivity.

**Springfield Vacant RES**

| Parcel | AV | sales price | Rank of AV | Rank of SP | Diff in Rank, sq |
|---|---|---|---|---|---|
| 650207327024 | 121300 | 112500 | 10 | 11 | 1 |
| 650207327026 | 121300 | 112500 | 10 | 11 | 1 |
| 650207328029 | 121300 | 82000 | 10 | 8 | 4 |
| 650207328030 | 151600 | 112500 | 12 | 11 | 1 |
| 650213400021 | 19800 | 18000 | 3 | 3 | 0 |
| 650233202002 | 9600 | 12500 | 1 | 2 | 1 |
| 650236100025 | 12300 | 11250 | 2 | 1 | 1 |
| 650606252013 | 69000 | 70000 | 7 | 6.5 | 0.25 |
| 650606276013 | 26400 | 26000 | 4 | 4 | 0 |
| 650207328032 | 151600 | 217500 | | | |
| 650207327025 | 141500 | | | | |
| | 293100 | 217500 | 13 | 13 | 0 |
| 650207328025 | 19000 | 66000 | | | |
| 650207328026 | 11400 | | | | |
| | 30400 | 66000 | 5 | 5 | 0 |
| 660207357031 | 32900 | 70000 | | | |
| 660207357032 | 33200 | | | | |
| | 66100 | 70000 | 6 | 6.5 | 0.25 |
| 650603429002 | 23100 | 93000 | | | |
| 650603429004 | 54800 | | | | |
| 650603429006 | 24700 | | | | |
| | 102600 | 93000 | 8 | 9 | 1 |
| | | Count | 13 | Sum | 10.5 |

Test Statistic = $\dfrac{1-(6*T)}{n*(n*n-1)}$   **-0.028**

The value is not significant.
No evidence of regressivity / progressivity.

# Spearman Rank Correlation Coefficient

## Critical Values for the Spearman Rank Correlation Coefficient

Two-Tail Test

| | 5 | 6 | 7 | 8 | 9 | 10 | **Numbers of Observations** | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 5% level | 1.000 | 0.886 | 0.786 | 0.738 | 0.700 | 0.648 | 0.618 | 0.587 | 0.560 | 0.538 | 0.521 | 0.503 | 0.488 | 0.474 | 0.460 | 0.447 |
| 1% level | na | 1.000 | 0.929 | 0.881 | 0.833 | 0.794 | 0.755 | 0.727 | 0.703 | 0.679 | 0.657 | 0.635 | 0.618 | 0.600 | 0.584 | 0.570 |

The statistic requires at least 5 observations to ascertain differences in ranks at the 95% confidence level.

# of observations greater than 20 - rely on PRD measure

# Improved Residential Comparison

### 2005 AV vs 2006 AV
### compared to 2004 to 2006 sales

| | WMW Test | Median Sold % increase | Median Unsold % Increase |
|---|---|---|---|
| Cass | -1.38 | 6.0 | 3.6 |
| Center | -1.714 | 7.7 | 7.4 |
| Clinton | 1.16 | 17.4 | 20.8 |
| Coolspring | -1.917 | 17.6 | 15.3 |
| Dewey | 0.34 | 9.4 | 11.8 |
| Galena | -0.43 | 22.4 | 23.2 |
| Hanna | 0.585 | 26.9 | 29.3 |
| Hudson | -2.017 | 15.7 | 12.8 |
| Johnson | -1.423 | 35.2 | 12.5 |
| Kankakee | -0.768 | 5.3 | 4.9 |
| Lincoln | -1.194 | 18.9 | 15.3 |
| Michigan | -4.562 | 28.6 | 22.0 |
| New Durham | 0.936 | 16.0 | 15.5 |
| Noble | 1.481 | 23.1 | 30.1 |
| Pleasant | 0.161 | 15.5 | 15.2 |
| Prairie | -0.456 | 21.4 | 20.5 |
| Scipio | -2.912 | 11.6 | 9.5 |
| Springfield | -0.026 | 17.8 | 19.2 |
| Union | -0.006 | 10.3 | 10.7 |
| Washington | -2.96 | 14.7 | 9.9 |
| Wills | -1.521 | 10.4 | 9.0 |

Confidence levels
90% + or - 1.645
95% + or - 1.96
99% + or - 2.58

# Michigan TWP Analysis

## Improved Residential Comparison

2005 AV vs 2006 AV
compared to 2004 to 2006 sales

|  | WMW Test | Median Sold % increase | Median Unsold % Increase |
|---|---|---|---|
| Township-wide | -4.562 | 28.6 | 22.0 |

## Lakefront vs. non-Lakefront

|  | WMW Test | Median Sold % increase | Median Unsold % Increase |
|---|---|---|---|
| Lakefront Neighborhoods | -0.145 | 83.46 | 85.9 |
| Non-Lakefront Areas | -4.529 | 21.8 | 17.0 |

* Lakefront Neighborhoods are: 160521, 160522, 410521, 410522,410523, 410533, 420503,420504, 4205041, 420512, 420519, 420521, 420522, 420553, 420554, 440521, 440522, 440534, 450520, 450521, 450522, 450589, 4205221

## Inner-City areas, vs. Lakefront vs. Condos / Tryon Farms

|  | WMW Test | Median Sold % increase | Median Unsold % Increase | % sold |
|---|---|---|---|---|
| Lakefront Neighborhoods | -0.145 | 83.46 | 85.9 | 10.6 |
| Inner-City Areas (all else) | -1.55 | 18.4 | 15.2 | 9.3 |
| Condos & Tryon Farms | -1.56 | 65.0 | 56.6 | 18.5 |

# Michigan TWP Non-Lake Neighborhoods: % change in AV 2005 to 06

| Nbrhd Nbr | # of parcels | # of Sales | % increase in Solds | % increase in UNSolds | |
|---|---|---|---|---|---|
| 42495 | 10 | 0 | na | 47 | |
| 420501 | 448 | 19 | 1.7 | 6.5 | |
| 420502 | 190 | 20 | 19.3 | 24.2 | |
| 420505 | 210 | 31 | 60 | 57.1 | |
| 420506 | 1035 | 86 | 15.2 | 14.7 | |
| 420507 | 214 | 19 | 2.5 | 3 | |
| 420508 | 680 | 54 | 22.7 | 22.8 | |
| 420510 | 16 | 2 | 40.9 | 41.7 | |
| 420511 | 52 | 1 | 10.3 | 12.9 | |
| 420513 | 134 | 8 | 9.5 | 4.9 | |
| 420514 | 151 | 9 | 52 | 51.8 | |
| 420515 | 125 | 12 | 21.4 | 21.9 | |
| 420516 | 186 | 21 | 3.4 | 0 | |
| 420518 | 40 | 11 | 23.2 | 22.7 | |
| 420520 | 15 | 0 | na | 54.1 | |
| 420524 | 591 | 51 | 11.4 | 10.4 | |
| 420527 | 45 | 5 | 102.1 | 89.4 | |
| 420528 | 8 | 2 | 94.8 | 95 | condos |
| 420529 | 102 | 10 | 8.5 | 10.3 | |
| 420530 | 40 | 10 | -12.4 | -8.5 | |
| 420531 | 335 | 46 | 22.4 | 21.8 | |
| 420532 | 538 | 48 | 13.4 | 10 | |
| 420533 | 106 | 7 | 6 | 5.3 | |
| 420534 | 485 | 47 | 29.5 | 29.3 | |
| 420535 | 249 | 18 | -1.7 | 5.3 | |
| 420536 | 440 | 40 | 5.9 | 3.9 | |
| 420537 | 630 | 68 | 5 | 4.8 | |
| 420538 | 104 | 3 | -4.8 | 0 | |
| 420540 | 4 | 0 | na | 43.8 | condos |
| 420542 | 88 | 2 | 2.3 | 1.6 | |
| 420543 | 64 | 6 | 52 | 52 | condos |
| 420544 | 11 | 2 | 104.1 | 107.4 | condos |
| 420545 | 9 | 5 | 55.3 | 51.8 | condos |
| 420546 | 4 | 1 | 77.4 | 77.4 | condos |
| 420547 | 22 | 8 | 24.2 | 23.8 | condos |
| 420548 | 8 | 1 | 60.9 | 60.9 | condos |
| 420549 | 47 | 8 | 31.4 | 34.9 | condos |
| 420550 | 76 | 10 | 31.8 | 31.9 | condos |
| 420551 | 15 | 1 | 119.8 | 135 | condos |
| 420552 | 37 | 11 | 80.7 | 80 | condos |
| 420553 | 1 | 0 | na | 190.8 | |
| 420555 | 39 | 4 | 83 | 99.1 | |
| 420558 | 17 | 6 | 243.7 | 169.6 | Tryon Farms |
| 420559 | 44 | 3 | 131.6 | 131.6 | condos |
| 420560 | 2 | 0 | na | 128.9 | condos |
| 420561 | 5 | 0 | na | -4.3 | condos |
| 420562 | 8 | 0 | na | 146.9 | condos |

| | | | | | |
|---|---|---|---|---|---|
| 420563 | 14 | 3 | 95.3 | 95.3 | condos |
| 420564 | 16 | 7 | 513.9 | 200.9 | Tryon Farms |
| 420565 | 1 | 1 | 443.1 | na | Tryon Farms |
| 420567 | 8 | 0 | na | 9.7 | condos |
| 420569 | 2 | 2 | 62.9 | na | |
| 420570 | 48 | 13 | 0 | 0 | |
| 420571 | 10 | 3 | 47.7 | 42.2 | |
| 420572 | 13 | 4 | 49.7 | 47.2 | |
| 420573 | 22 | 4 | 200 | 200 | boat slips |
| 420575 | 10 | 1 | 85.1 | 83.1 | condos |
| 420577 | 4 | 4 | 74.8 | na | condos |
| 420580 | 5 | 2 | 273 | 358.4 | Tryon Farms |
| 420583 | 32 | 4 | 55.8 | 56.6 | condos |
| 420587 | 1 | 1 | 102 | 102 | |
| 420591 | 2 | 1 | 42.9 | 77.1 | condos |
| 430510 | 83 | 4 | 61.4 | 47.2 | |
| 440535 | 2 | 0 | na | 29.9 | |
| 460512 | 559 | 63 | 17.8 | 17 | |
| 460513 | 15 | 3 | 10.9 | 11.8 | |
| 470510 | 25 | 3 | 27.5 | 46.2 | |
| 470588 | 2 | 0 | na | 46.2 | |
| 500512 | 6 | 1 | 12.8 | 19.3 | |
| 4205271 | 4 | 0 | na | 22.9 | |
| 4205281 | 11 | 4 | 51 | 55.5 | condos |
| 4205282 | 12 | 3 | 45.4 | 45.4 | condos |
| 4205283 | 36 | 7 | 56.9 | 56.9 | condos |
| 4205284 | 1 | 0 | na | 75.1 | condos |
| 4205285 | 4 | 1 | 16 | 149.9 | condos |
| 4205461 | 19 | 4 | 45.1 | 65.2 | condos |
| 4205462 | 13 | 1 | 8.6 | 11 | condos |
| 4205463 | 4 | 0 | na | 29.1 | condos |
| 4205601 | 2 | 1 | 126.1 | 126.1 | condos |
| 4205602 | 2 | 0 | na | 112.1 | condos |
| 4205603 | 2 | 0 | na | 113.5 | condos |
| 4205631 | 14 | 2 | 87.6 | 87.6 | condos |
| 4205632 | 14 | 2 | 80.1 | 80.1 | condos |
| 4205633 | 15 | 3 | 98.9 | 98.9 | condos |

| **Lakefront Neighborhoods** | # of Parcel | # sold | median<br>% increase SOLD | median<br>% increase UNSOLD |
|---|---|---|---|---|
| 160521 | 1 | 0 | na | 122.6 |
| 160522 | 69 | 10 | 50.2% | 56.3% |
| 410522 | 31 | 1 | 45.6% | 50.3% |
| 410523 | 145 | 15 | 81.2% | 79.8% |
| 410533 | 14 | 4 | 103.9% | 61.0% |
| 420503 | 102 | 10 | 83.8% | 112.6% |
| 420504 | 80 | 17 | 66.0% | 58.3% |
| 420512 | 1 | 0 | na | 39.7 |
| 420519 | 201 | 30 | 91.2% | 75.7% |
| 420521 | 97 | 6 | 115.6% | 112.3% |
| 420522 | 49 | 6 | 64.9% | 94.1% |
| 420523 | 31 | 2 | 34.6% | 10.2% |
| 420553 | 11 | 2 | 56.6% | 52.0% |
| 420554 | 20 | 2 | 118.3% | 87.8% |
| 440522 | 9 | 0 | na | 52.9% |
| 440534 | 209 | 15 | 101.9% | 108.5% |
| 450520 | 722 | 70 | 85.1% | 87.9% |
| 450521 | 128 | 11 | 86.3% | 81.6% |
| 450522 | 163 | 16 | 85.4% | 80.9% |
| 450589 | 5 | 0 | na | 281.8% |
| 4205041 | 15 | 6 | 104.0% | 91.6% |
| 4205221 | 13 | 1 | 120.6% | 82.6% |

reasonable in circumstances where the precise population distribution is in doubt.

*Computational aspects.* Given a program for the Fisher exact test for a 2×2 contingency table one can easily determine for any given $m$, $n$ the number above the median in the first sample which just gives significance, and, because there is only one degree of freedom, all other entries in the 2×2 table. It is then, as indicated in the above example, relatively simple to determine confidence limits by appropriate additions or subtractions from all second-sample observations.

### 5.1.3 The Wilcoxon–Mann–Whitney test

The literature refers to equivalent tests formulated in different ways as the **Wilcoxon rank sum test** and the **Mann–Whitney test**. The formulations were developed independently by Wilcoxon (1945) and Mann and Whitney (1947). We refer to the two versions jointly as the **Wilcoxon–Mann–Whitney test** or, for brevity, as the **WMW test**. Example 1.4 gave a specific case of the Wilcoxon formulation that reflected the basic theory directly. The Mann–Whitney approach is easy to apply and leads more directly to confidence intervals for differences between population means or medians.

The Wilcoxon formulation needs a joint ranking of observations from the two samples, and we sum the ranks associated with one sample. As indicated in Example 1.4, if both samples come from the same population (which may be of any continuous form and need not be symmetric) we expect a mix of low, medium and high ranks in each sample. If the alternative to the null hypothesis of identical populations is that the population distributions differ only in location (i.e. mean or median), then under that alternative we expect lower ranks to dominate in one population and higher ranks in the other. A shift in location epitomizes the concept of an 'additive' treatment effect, or a 'constant' difference between two treatments. The test is also relevant when we sample from two distributions with cumulative distribution functions $F(u)$ and $G(v)$ identical under $H_0$, but under $H_1$, for all $x$, either $F(x) \leq G(x)$ or $F(x) \geq G(x)$ with strict inequality for at least some $x$; a moment's reflection shows that under $H_1$ low or high ranks should dominate in one sample, as opposed to a fairly even distribution of ranks under $H_0$. Given the permutation distribution of rank sums under $H_0$, critical regions may be determined in the way described for the particular case in Example 1.4.

*Example 5.3*

*The problem.* Given the data on page numbers for books on biology and management in Example 5.2, test the hypothesis that the medians do not differ against a two-sided alternative. The data are